PARISHRAM 2024

- FOR 12TH CLASS CBSE BOARD STUDENTS -

- Subject Name- Mathematics
- Chapter Name- Relations and Functions

Lecture No.- 07

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Today's Targets

1. One-One Function
2. Onto Function
3. Even Function
4. Odd Function

Types of Functions

$\forall \Rightarrow$ for all / for every

$\exists \Rightarrow$ there exists

One-One Function

- Every $x \in A$ must have unique image
- $n(A) \leq n(B)$
- Range $\subseteq$ Co-domain

$$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$

Onto Function

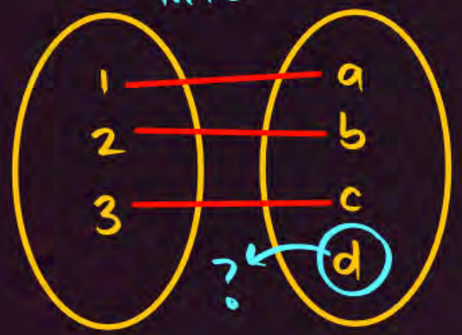
- Every $y \in B$ must have at least one pre-image in A
- $n(B) \leq n(A)$
- Range = Co-domain

$$\forall\, y \in B\ \exists\, x \in A : y = f(x)$$

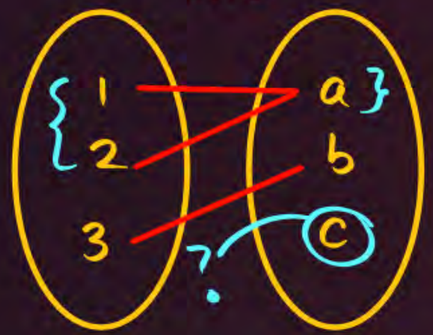
Bijective Function

- Both one-one and onto
- $n(A) = n(B)$
- Range = Co-domain

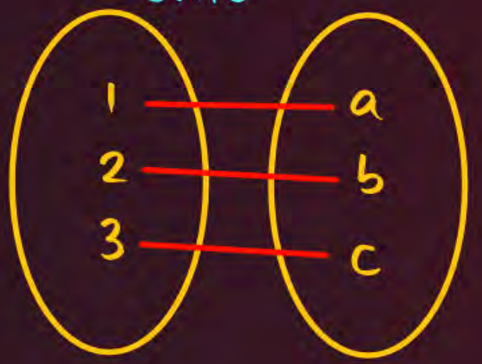
One - One
+
into



Many - One
+
into



One - One
+
onto



Types of Functions

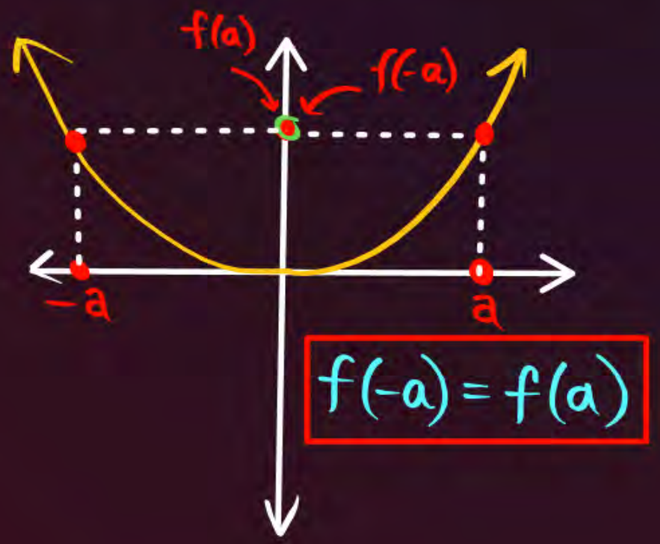
Even Functions

$$f(-x) = f(x)$$

$$f(-x) = \begin{cases} f(x), & f \text{ is even} \\ -f(x), & f \text{ is odd} \end{cases}$$

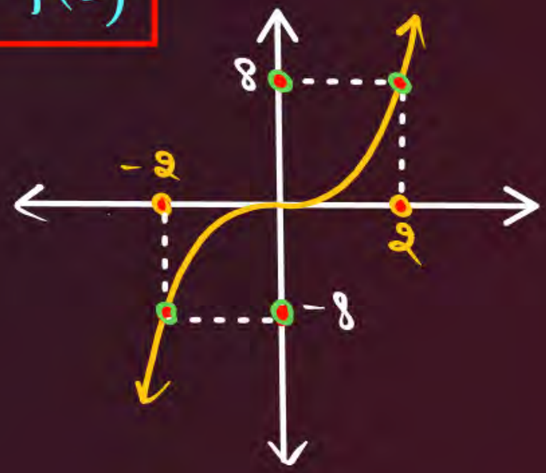
Odd Functions

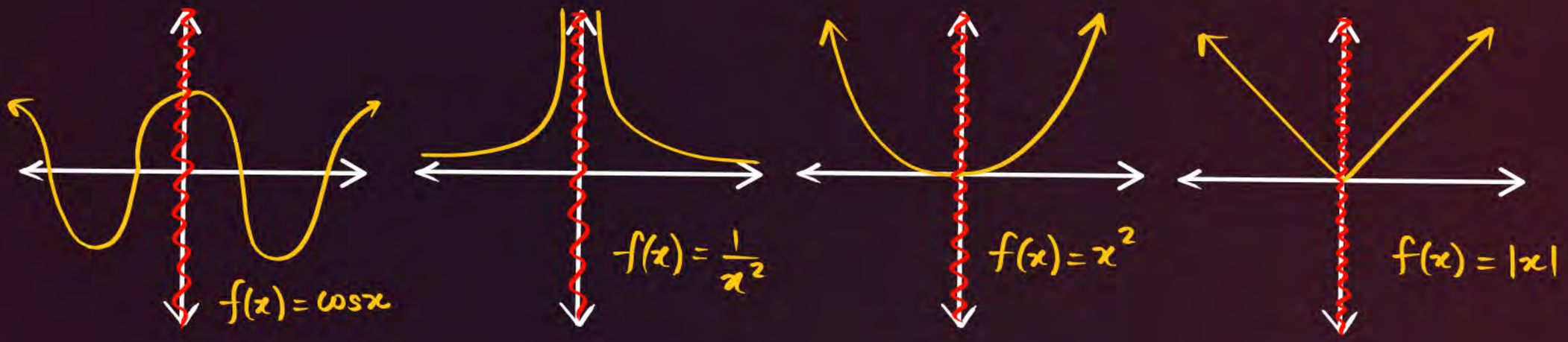
$$f(-x) = -f(x)$$

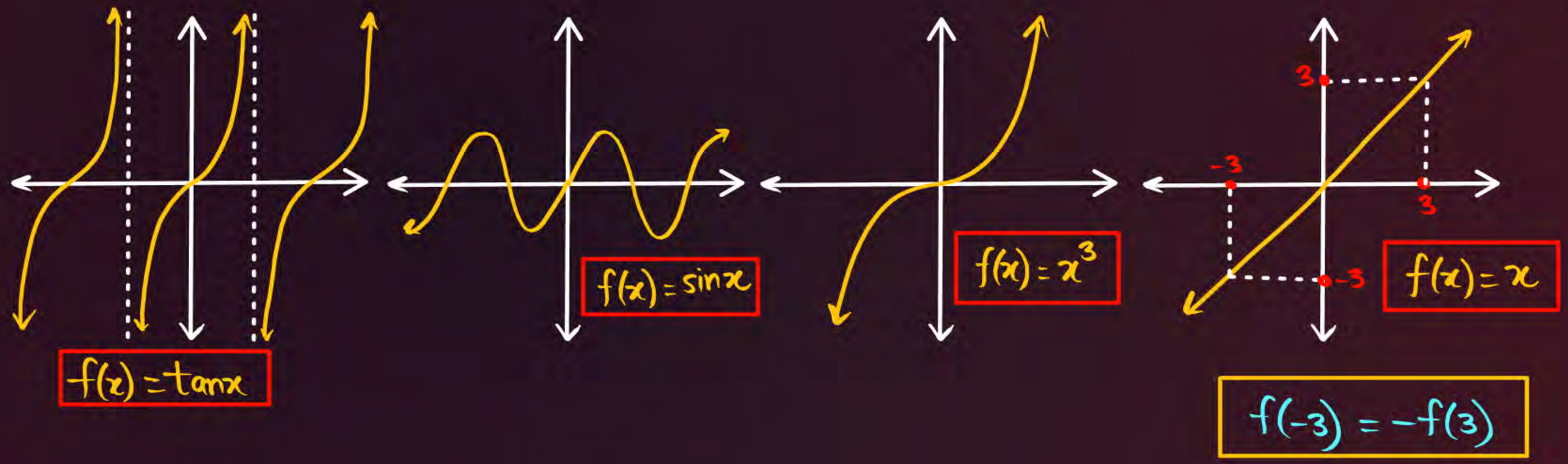


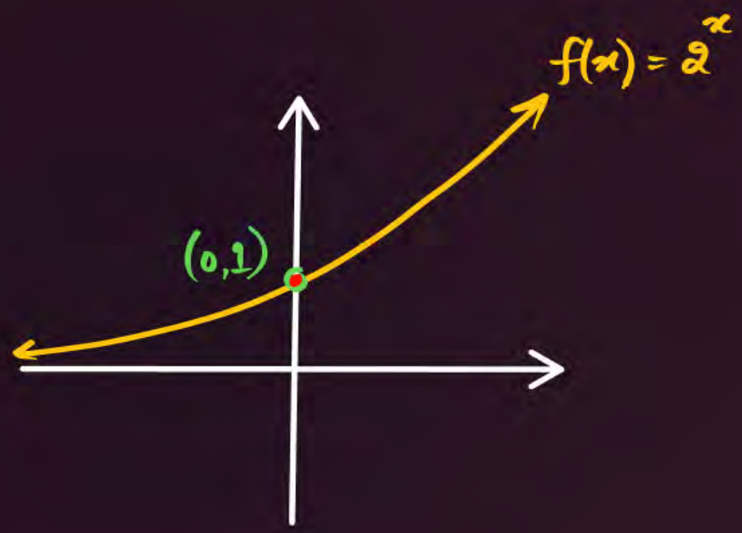
$f(-a) = f(a)$

$f(-2) = -f(2)$




f(x) = cos x
f(x) = 1/x²
f(x) = x²
f(x) = |x|


f(x) = tanx
f(x) = sinx
f(x) = x^3
3
-3
3
-3
f(x) = x
f(-3) = -f(3)



Neither even nor odd.

$$f(x) = (2)^x$$

at $x = 1$

$$f(1) = (2)^1 = 2$$

at $x = -1$

$$f(-1) = (2)^{-1} = \frac{1}{2}$$

Question

one-one ness ↗ ↗ onto ness

$N = \{1, 2, 3, 4, 5, \ldots\}$

Check the injectivity and surjectivity of the following functions:

(i) $f : N \to N$ given by $f(x) = x^2$

(ii) $f : Z \to Z$ given by $f(x) = x^2$

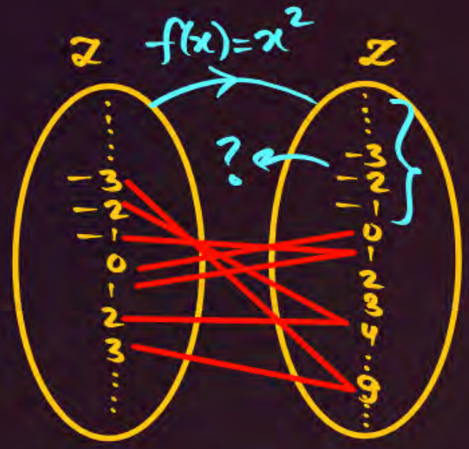
(iii) $f : R \to R$ given by $f(x) = x^2$

(iv) $f : N \to N$ given by $f(x) = x^3$

(v) $f : Z \to Z$ given by $f(x) = x^3$

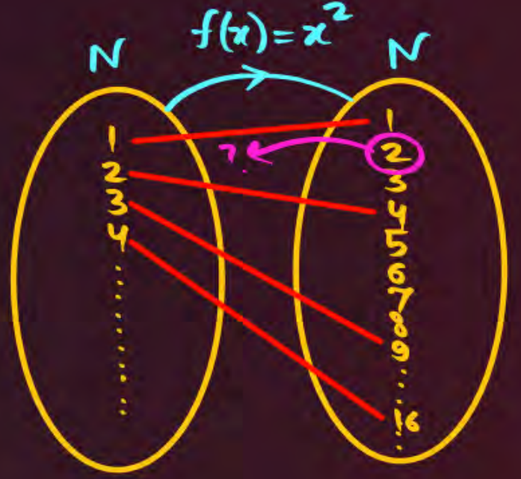
} HW

(ii) $f : Z \to Z$, $f(x) = x^2$



f is not one-one
because $f(1) = f(-1) = 1$
f is not onto
because there is no such value of $x \in Z$ such that $x^2 = -1$

(i) $f : N \to N$, $f(x) = x^2$



f is one-one
but no such value of $x \in N$ such that $x^2 = 2$.
$\Rightarrow$ f is not onto.

$a^3-b^3=(a-b)(a^2+ab+b^2)$

(iii) $f: R \to R$, $f(x)=x^3$

Ontoness

Let $y \in R$ such that $y=f(x)=x^3$

$y=x^3$

$\Rightarrow x=y^{1/3}$

Cube of real no. = real no.

$y \in R \Rightarrow y^{1/3} \in R$

$\Rightarrow x \in R$

$\Rightarrow$ f is onto.

One-one ness.

Let $f(x_1)=f(x_2)$

$\Rightarrow x_1^3=x_2^3$

$\Rightarrow x_1^3-x_2^3=0$

$\Rightarrow (x_1-x_2)(x_1^2+x_1x_2+x_2^2)=0$

$x_1-x_2=0$ or $x_1^2+x_1x_2+x_2^2=0$

$\Rightarrow x_1=x_2$

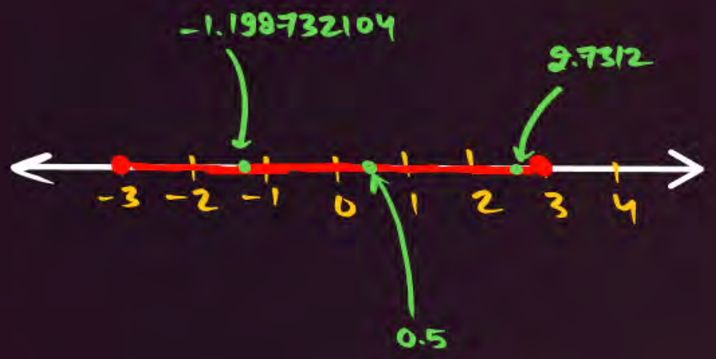
only when

$\Rightarrow x_1=x_2=0$

$\Rightarrow$ f is one-one

Interval

$[-1, 1]$



Set

$A = \{0, 1, 2, 3, 4, 5, \ldots\}$

discrete elements

Question

$R_* = R - \{0\}$

Show that the function $f: R_* \to R_*$ defined by $f(x) = \frac{1}{x}$ is one-one and onto, where R_* is the set of all non-zero real numbers. Is the result true, if the domain R_* is replaced by N with co-domain being same as R_*?

Sol:

One-One ness

Let $f(x_1) = f(x_2)$

$\Rightarrow \frac{1}{x_1} = \frac{1}{x_2}$

$\Rightarrow x_2 = x_1$

$\Rightarrow$ f is one-one

ontoness

Let $y \in R_*$ such that

$y = \frac{1}{x} \Rightarrow x = \frac{1}{y}$

$y \in R_* \Rightarrow \frac{1}{y} \in R_* \Rightarrow x \in R_*$

$\Rightarrow$ f is onto

(ii) $f: N \to R_*$

$f(1) = \frac{1}{1} = 1 \in R_*$

$f(2) = \frac{1}{2} \in R_*$

$f(3) = \frac{1}{3} \in R_*$

$f(4) = \frac{1}{4} \in R_*$

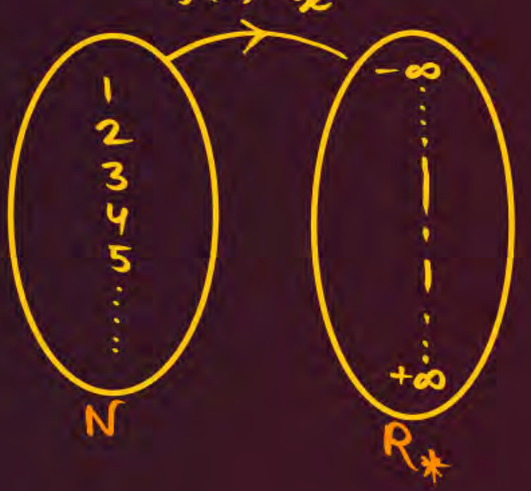
$f(n) = \frac{1}{n} \in R_*$

$\Rightarrow$ f is one-one

But $y = -1 \in R_*$

there is not such element $x \in N$ such that $-1 = \frac{1}{x}$.

$\Rightarrow$ f is not onto.



Question

Prove that the Greatest Integer Function $f : R \to R$, given by $f(x) = [x]$, is neither one-one nor onto, where $[x]$ denotes the greatest integer less than or equal to x.

Sol: $f(x) = [x]$. $f : R \to R$

(i) One-one ness

$[2.5] = 2$

$[2.4] = 2$

$\therefore f(2.5) = f(2.4)$

$\Rightarrow$ f is not one-one

(ii) Onto ness

let $y = 2.5 \in R$

but there is no such value of $x \in R$ such that

$[x] = 2.5$

$\Rightarrow$ f is not onto.

Hint

$[x] \in Z$

$[1.5] = 1 \in Z$

$[2.3] = 2 \in Z$

$[-1.5] = -2 \in Z$

Question

Show that the Modulus Function $f : R \to R$, given by $f(x) = |x|$, is neither one-one nor onto, where $|x|$ is x, if x is positive or 0 and $|x|$ is $-x$, if x is negative.

Sol. $f : R \to R$, $f(x) = |x|$

(i) One-one ness

$f(2) = |2| = 2$

$f(-2) = |-2| = 2$

$f(2) = f(-2) = 2$

$\Rightarrow$ f is not one-one

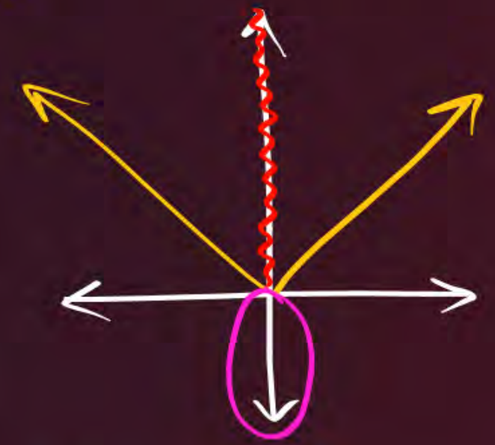
(ii) onto ness

Let $y = -2 \in R$

there is no such value of $x \in R$ such that

$|x| = -2$.

$\Rightarrow$ f is not onto.



Range $= [0, \infty)$

Codomain $= (-\infty, \infty)$

Question

Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. Show that f is one-one.

Sol: $f : A \to B$, $f = \{(1,4), (2,5), (3,6)\}$

$f(1) = 4$

$f(2) = 5$

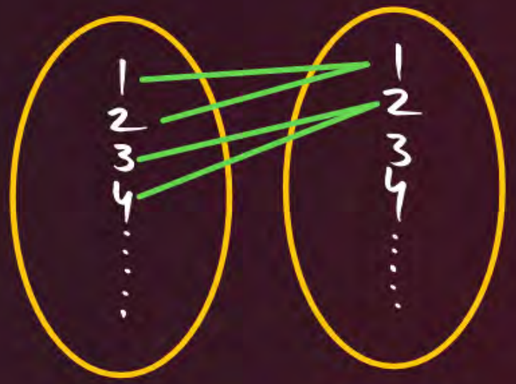
$f(3) = 6$

Every $x \in A$ has its unique image in B

$\Rightarrow$ f is one-one

Question

Let $f: N \to N$ be defined by $f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$ for all $n \in N$. State whether the function is bijective. Justify your answer.



Sol: (i) One-one ness.

$f(1) = \frac{1+1}{2} = 1$

$f(2) = \frac{2}{2} = 1$

$f(1) = f(2) = 1$

$\Rightarrow$ f is not one-one

$\Rightarrow$ f is not bijective.

Question

HW

Show that the Signum function $f: R \to R$, given by

$$f(x) = \begin{cases} 1, \text{if } x > 0 \\ 0, \text{if } x = 0 \\ -1, \text{if } x < 0 \end{cases}$$ is neither one-one nor onto.

Question

V. Imp. (HW)

Let $A = R - \{3\}$ and $B = R - \{1\}$. Consider the function $f: A \to B$ defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.

THANK
YOU